Black Hole thermodynamics without a black hole?

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Abstract

In the present paper we consider, using our earlier results, the process of quantum gravitational collapse and argue that there exists the final quantum state when the collapse stops. This state, which can be called the "no-memory state", reminds the final "no-hair state" of the classical gravitational collapse. Translating the "no-memory state" into classical language we construct the classical analogue of quantum black hole and show that such a model has a topological temperature which equals exactly the Hawking's temperature. Assuming for the entropy the Bekenstein-Hawking value we develop the local thermodynamics for our model and show that the entropy is naturally quantized with the equidistant spectrum $S + \gamma_0 N$. Our model allows, in principle, to calculate the value of γ_0. In the simplest case, considered here, we obtain $\gamma_0 = \ln 2$.

Preliminaries

In 1972 J.D.Bekenstein observed the striking resemblance of the Schwarzschild black hole mechanics with the first and second laws of thermodynamics [1]. He presented very serious physical arguments that the Schwarzschild black hole should be ascribed by a certain amount of entropy which is proportional to the event horizon area. In 1973 J.M.Bardeen, B.Carter and S.W.Hawking extended this idea and proved the four laws of thermodynamics for the general class of Kerr-Newman black hole [2], the role of the temperature being played by the surface gravity (up to some numerical factor), which is constant along the event horizon. And only after discovering by S.W.Hawking the black hole evaporation [3] this analogy became the real physical phenomenon. It appeared that the spectrum of such a radiation is Planckian with the temperature $T_H = \frac{\kappa}{2\pi}$, where κ is the surface gravity. It follows then that the black hole entropy is exactly one fourth of the dimensionless horizon area,

$$S = \frac{1}{4}\frac{A}{l_{Pl}^2}, \tag{1}$$

where $l_{Pl} = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-33} cm$ is the Planckian length ($\hbar$ is the Planck constant, c is the speed of light, and G is the Newtonian gravitational constant). We use the units $\hbar = c = k = 1$ (k is the Boltzmann constant), so $l_{Pl} = \sqrt{G}$ and the Planckian mass is $m_{Pl} = \sqrt{\frac{\hbar c}{G}} = 1/\sqrt{G} \sim 10^{-5} gr$.

The nature of such a radiation and its black body spectrum lies in the nontrivial causal structure of the space-times containing black holes. The crucial point is the existence of the horizons. The same takes place in the Rindler spacetime which is actually the part of the flat Minkowski spacetime with the event horizon. The Rindler's observer experiences a constant acceleration a and "sees" a thermal bath with the temperature $\frac{a}{2\pi}$ which is called the Unruh's temperature [4]. The Hawking's temperature T_H is just the Unruh's temperature T_U at the event horizon.

The quantum nature of the radiation implies the quantization of the black hole mass. The first attempt was made by J.D.Bekenstein [5]. He noticed that the horizon area of non-extremal black holes behaves as a classical adiabatic invariant. The Bohr-Sommerfeld quantization rule then predicts the equidistant discrete spectrum for the horizon area and, thus, for the black

hole entropy. The gedanken experiments show that the minimal increase in the horizon area in the process of capturing the neutral [6] or electrically charged [7] particle is approximately equals to

$$\Delta A_{min} \approx 4 l_{Pl}^2. \tag{2}$$

This suggests for the black hole entropy (provided we accept the relation (1))

$$S_{BH} = \gamma_0 N, \qquad N = 1, 2, ... \tag{3}$$

where γ_0 is of order of unity. In their famous work on the black hole spectroscopy J.D.Bekenstein and V.F.Mukhanov [8] related the black hole entropy to the number of microstates g_n that corresponds to a particular external macrostate through the well known formula in statistical physics, $g_n = \exp\left[S_{BH}(n)\right]$, e.i., g_n is the degeneracy of the nth area eigenvalue. Since g_n should be an integer they deduce that

$$\gamma_0 = \ln k, \qquad k = 2, 3, ... \tag{4}$$

In the spirit of the information theory and "It from Bit"-idea by J.A.Wheeler the value of $\ln 2$ seems the most suitable one. The equidistant area spectrum was also derived from the some symmetry principles [9, 10, 11].

The confirmation of microscopical statistical nature of the black hole entropy came from the string theory. A.Strominger and C.Vafa [12] were the first who counted directly the degeneracy of the horizon microstates in the special case of 5-dimensional extremal black hole and showed that the relation (1) is exact. A review of further progress can be found in [13].

The very natural way of counting the number of microscopic states is provided by the Loop Quantum Gravity (see [14] for a recent review). In this rather new approach to canonical quantization of gravity the area operator has a discrete spectrum. Such an operator can be represented by a spin network puncturing a surface. The procedure of counting the surface states at the horizon was developed by K.Krasnov [15] and applied to calculating the black hole entropy by A.Ashtekar et al. [16, 17]. The net result is that the entropy of the spherically symmetric black hole equals

$$S_{BH} = N \ln\left(2 j_{min} + 1\right), \tag{5}$$

where j_{min} is the minimal (nonzero) spin value depending on the underlying symmetry group. In the conventional Loop Quantum Gravity this is the

$SU(2)$ group, thus, $j_{min} = \frac{1}{2}$, and for γ_0 (Eqn.(4)) we have $\gamma_0 = \ln 2$. For the $SO(3)$ group $j_{min} = 1$, and $\gamma_0 = \ln 3$. For the horizon area Loop Quantum Gravity gives in this case the value

$$A_h = 8\pi\gamma\sqrt{j_{min}(j_{min}+1)}Nl_{Pl}^2, \tag{6}$$

where γ is the so-called Immirzi ambiguity parameter [18]. It equals $\frac{\ln 2}{\sqrt{3}\pi}(\approx 0.12738402)$ for $j_{min} = \frac{1}{2}$ and $\frac{\ln 3}{2\sqrt{2}\pi}(\approx 0.12363732)$ for $j_{min} = 1$, provided $S_{BH} = \frac{1}{4}\frac{A_h}{l_{Pl}^2}$. Thus, Loop Quantum Gravity gives us almost unique (up to the choice of j_{min} and, of course for large N) quantum spectrum for the black hole entropy, but the horizon ares and, hence, the black hole mass spectra depend on the choice of the Immirzi parameter.

The recent progress in this subject is connected to the so-called quasi-normal modes of the Schwarzschild black hole. It is known for a long time that the decay of black hole perturbations is dominated at late times by a set of damped oscillations (see, e.g. [19]). It was shown that for the frequencies ω with large imaginary part, the real part becomes equally spaced, and

$$m\omega = 0.04371235 + \frac{i}{4}(n + \frac{1}{2}), \tag{7}$$

where m is the black hole mass [20, 21]. S.Hod noticed [22] that the real part of ω can be written as (and it was later proven analytically [23])

$$\omega_{QNM} = \frac{\ln 3}{8\pi m}. \tag{8}$$

The Bohr's correspondence principle requires $dm = \omega_{QNM}$, and for the entropy we obtain $S_{BH} = N \ln 3$.

In all the above mentioned approaches to quantizing the black hole area (or the entropy content) the event horizon is considered essentially classical. But, in quantum theory there are no trajectories, no geodesics to probe the spacetime geometry, so, the very notion of the event horizon is not defines. Therefore, there exists no definition of what a quantum black hole is.

To overcome this difficulty we construct some very simple classical model (namely, the self-gravitating dust shell), then quantize it using minisuperspace formalism and try to extract some physical information [24, 25, 26, 27]. In the present paper we give a short outline of the classical model, the quantization procedure and the resulting mass spectra. Then we argue that the

very process of quantum gravitational collapse gives rise to the increase of entropy (which is initially zero). The final stage of the quantum collapse is a special "no-memory" quantum state that resembles the black hole "no-hair" feature. We named it "a quantum black hole". At the end of the paper we show that it is possible to construct the classical analogue of such a quantum state. This classical analogue possesses a topological temperature which coincides exactly with the Hawking's temperature for the Schwarzschild black hole. We give also a complete thermodynamical description of the model, derive the equidistant area (and entropy) spectrum and show how the entropy units can, in principle, be calculated.

Classical Model

Everybody knows what the classical black hole is. In short, black hole is a region of a space-time manifold beyond an event horizon. In turn, an event horizon is a null surface that separates the region from which null geodesics can escape to infinity and that one from which they cannot. It is important to stress that the notion of the of the event horizon is global, it requires knowledge of both past and future histories. In classical physics we have trajectories of particles, we have geodesics, so, everything can be, in principle, calculated. In quantum physics there are no trajectories and the event horizon can not be defined. Thus, we have to seek for quite a different definition of a quantum black hole. Till now we have no consistent theory of quantum gravity. All this forces us to start with considering some models. The simpler, the better.

The simplest is the so-called Schwarzschild eternal black hole. Its geometry is a geometry of non-traversable wormhole. There are two asymptotically flat regions at spatial infinities connected by the Einstein-Rosen bridge. The gravitating source is concentrated at two space-like singular surfaces or zero radius. Two sides of the Einstein-Rosen bridge are causally disconnected and separated by event horizons. The narrowest part of the bridge is called a throat, its size is the size of the horizon. Eternal black holes are parameterized by total (Schwarzschild) mass of the system. This one-parameter family is the only spherically symmetric solution to the vacuum Einstein equations. The spherically symmetric gravity can be fully quantized in the minisuperspace (frozen) formalism [28, 29]. The result of such quantization is trivial, quantum functional depends only on Schwarzschild mass. Physically it is quite understandable. Indeed, one allows the matter sources first to collapse classically and then starts to quantize such a system. What is left for quantization? Nothing. Mathematically, eternal black holes has no dynamical degrees of freedom. No real gravitons (because of frozen spherical symmetry), no matter source motion.

To get physically meaningful result we need to introduce some dynamical gravitating source. The simplest generalization of the point mass is the spherically symmetric self-gravitating thin dust shell. The theory of thin shells was developed by W.Israel [30] and applied to various problems by many authors. For simplicity we consider the case when the shell is the only

source of gravitational field. Then, inside the shell the space-time is flat, and outside it is some part of Schwarzschild solution. The dynamics of such dust shell is completely described by the single equation [31]

$$\sqrt{\dot{\rho}^2+1}-\sigma\sqrt{\dot{\rho}^2+1-\frac{2Gm}{\rho}}=\frac{GM}{\rho} \tag{9}$$

where ρ is the radius of the shell as a function of proper time of an observer sitting on the shell, a dot denotes the proper time derivative, m is the total (Schwarzschild) mass of the shell, and M is the bare mass (e.g., the sum of the masses of constituent dust particles without gravitational mass defect). The quantify σ is the sign function distinguishing two different types of shells. If $\sigma=+1$, the shells moves on "our" side of the Einstein-Rosen bridge and the radii increase when one goes in the outward direction of the shell. We will call this the black hole case. If $\sigma=-1$, the shell moves beyond the event horizon on the other side or the Einstein-Rosen bridge, and radii out of the shell first start to decrease, reach the minimal value at the throat and start to increase already on our side of the bridge. We will call this the wormhole-like case (such a configuration is also called a semi-closed world). In what follows we confine ourselves by considering the bound motion only. It can be shown that

$$\begin{aligned}\frac{m}{M}&>\frac{1}{2}\qquad if\qquad \sigma=+1\\ \frac{m}{M}&<\frac{1}{2}\qquad if\qquad \sigma=-1\end{aligned} \tag{10}$$

The two types of shells can be distinguished by different signs of the following inequality (ρ_0 is the radius of the shell at the turning point)

$$\begin{aligned}\frac{\partial m}{\partial M}&>0\qquad if\qquad \sigma=+1\\ \frac{\partial m}{\partial M}&<0\qquad if\qquad \sigma=-1\end{aligned} \tag{11}$$

The seemingly unusual sign in the wormhole case can be easily explained. Indeed, the large the bare mass M of the shell, the stronger its gravitational field, the more narrow, therefore, the throat, and, consequently, the smaller the total mass m of the system.

Quantum Model

The spherically symmetric space-times with shells can also be fully quantized in the minisuperspace formalism [26]. All the quantum constraints can be solved, except one. This is the Hamiltonian constraint or, Wheeler-DeWitt equation, for the shell (here we write it only for the case of bound motion)

$$\Psi(s + i\zeta) + \Psi(s - i\zeta) = \frac{2 - \frac{1}{\sqrt{s}} - \frac{M^2}{4m^2 s}}{(1 - \frac{1}{\sqrt{s}})^{1/2}} \Psi(s) \tag{12}$$

Here s is a dimensionless radius squared (normalized by the horizon area, $s = R^2/R_g^2 = R^2/4G^2m^2$), $\zeta = \frac{1}{2}(\frac{m_pl}{m})^2$, and i is the imaginary unit. The Eqn.(12) is an equation in finite differences, and the shift in the argument is pure imaginary. Thus, the "good" solutions should be analytical functions. Besides, there are branching points at the horizons (in our case at $s = 1$). Thus, the wave functions should be analytical on a Riemann's surface with a two leaves. The physical reason to consider two Riemann's surface is the following. In quantum theory there are no trajectories. Thus, even if a shell has parameters m and M (total and bare mass) corresponding to the black hole (or wormhole) case, its wave function is, in general, everywhere nonzero, "feel" both infinities on both sides of Einstein-Rosen bridge. The analyticity requirement is so stringent that there is no need to solve the quantum equation in order to calculate a mass spectrum. One should investigate only a behavior of solutions in the vicinity singular points (infinities and singularities) and around branching points, and then to compare these asymptotics. In such a way the following quantum conditions were found for a discrete mass spectrum in the case of bound motion [26].

$$\begin{gathered} \frac{2m^2 - M^2}{\sqrt{M^2 - m^2}} = \frac{2m_{pl}^2}{m} n \\ M^2 - m^2 = 2m_{pl}^2(1 + 2p) \end{gathered} \tag{13}$$

where n and p are integers. The appearance of two quantum conditions instead of only one in conventional quantum mechanics is due to a nontrivial causal structure of Schwarzschild manifold (two infinities!).

Let us discuss some properties of the spectrum that arises from these conditions.

1. For larger values of quantum number n ($\frac{M^2}{m^2} - 1 << 1$) one can easily derive nonrelativistic Rydberg formula for Kepler's problem, $E_{nonrel} = M - m = -\frac{G^2 M^4}{8n^2}$.

2. The role of turning point ρ_0 is now played by the integer n. Thus, keeping n constant and calculating $\gamma = \frac{\partial m}{\partial M}|_n$ one can distinguish between a black hole case ($\gamma > 0$) and a wormhole case ($\gamma < 0$). It appears that $\frac{\partial m}{\partial M}|_n > 0$ for $n \geq n_0$, negative or zero, and

$$|n_0| = E[\sqrt{2}\sqrt{13\sqrt{5} - 29}(1 + 2p)] \tag{14}$$

3. There exists a minimal possible value for a black hole mass. This occurs if $p = n_0 = 0$,

$$m_{min} = \sqrt{2} m_{pl} \tag{15}$$

4. The spectrum described by Eqn.(14) is not universal in the sense that corresponding wave functions form a two-parameter family $\Psi_{n,p}(R)$.

But for quantum Schwarzschild black hole we expect a one-parameter family of wave functions. Quantum black holes should have no hairs, otherwise there will be no smooth limit to the classical black holes. All this means that our spectrum is not a quantum black hole spectrum, and our shell does not collapse (like an electron in hydrogen atom). Physically, it is quite understandable, because the radiation is yet included into consideration.

And again, we will use thin shells to model the radiation, but this time shells should be null. Let m_{in} and m_{out} be a Schwarzschild mass inside and outside the shell. Then, the quantum constraint equation reads as follows [27]

$$\Psi(m_{in}, m_{out}, s - i\zeta) = \sqrt{\frac{1 - \frac{\mu}{\sqrt{s}}}{1 - \frac{1}{\sqrt{s}}}}\Psi(m_{in}, m_{out}, s) \tag{16}$$

here $\mu = m_{in}/m_{out}$, $\zeta = \frac{1}{2} m_{pl}^2 / m_{out}^2$. The existence of the second infinity on the other side of the Einstein-Rosen bridge leads to the following quantization condition ($m = m_{out}$)

$$\delta m = m_{out} - m_{in} = -2m + 2\sqrt{m^2 + k m_{pl}^2}, \tag{17}$$

where k is an integer. It is interesting to note that if we put $k = 1$ (minimal radiating energy) and require $\delta m < m$ (not more than the total mass can be

radiated away), then we obtain

$$m = m_{out} > \frac{2}{\sqrt{5}} m_{pl}. \tag{18}$$

Thus, the black hole with the mass given by Eqn.(15) is not radiating and, therefore, it can not be transformed into semi-closed world (wormhole-like case).

The discrete spectrum of radiation (17) is universal in the sense that it does not depend on the structure and mass spectrum of the gravitating source. This means that the energies of radiating quanta do not coincide with level spacing of the source. The most natural way in resolving such a paradox is to suppose that quanta are created in pairs. One of them is radiated away, while another one goes inside. Thus, the quantum collapse can not proceed without radiating even in the case of spherical symmetry. This radiation is accompanying with creation of new shells inside the primary shell we started with. We see, that the internal structure of quantum black hole is formed during the very process of quantum collapse. And if at the beginning we had one shell and knew everything about it, then already after the first pulse of radiation we have more than one way of creating the inner quantum. So, initially the entropy of the system was zero, it starts to grow during the quantum collapse. If somehow such a process would stop we would call the resulting object "a quantum black hole". The natural limit is the transition from black hole to the wormhole-like shell. The matter is that such a transition requires (at least in quasi-classical regime) insertion of an infinitely large volume, and the quasi-classical probability for this process is zero.

Let us write down the spectrum of the shell with nonzero Schwarzschild mass, the total mass inside, $m_{in} \neq 0$

$$\frac{2(\Delta m)^2 - M^2}{\sqrt{M^2 - (\Delta m)^2}} = \frac{2m_{pl}^2}{\Delta m + m_{in}} n \tag{19}$$

$$M^2 - (\Delta m)^2 = 2(1+2p) m_{pl}^2$$

Here Δm is the total mass of the shell, M is the bare mass, the total mass of the system equals $m = m_{out} = \Delta m + m_{in}$. For the black hole case $M^2 < 4m\Delta m$, or

$$\frac{\Delta m}{M} > \frac{1}{2}\left(\sqrt{(\frac{m_{in}}{M})^2 + 1} - \frac{m_{in}}{M}\right). \tag{20}$$

After switching on the process of radiation governed by Eqn.(17), the quantum collapse starts. Our computer simulations shows that evolves in the "correct" direction, e.g. it becomes nearer and nearer to the threshold (20) between the black hole case and wormhole case. The process stops exactly at $n = 0$!

The point $n = 0$ in the spectrum is very special. Only in such a state the shell does not "feel" not only the outer regions (what is natural for the spherically symmetric configuration) but it does not know anything about what is going on inside. It "feel" only itself. Such a situation reminds the classical (non-spherical) collapse. Finally when all the shells (both the primary one and newly produced) are in the corresponding states $n_i = 0$, the system does not "remember" its own history. And this is a quantum black hole. The masses of all the shells obey the relation

$$\Delta m_i = \frac{1}{\sqrt{2}} M_i. \qquad (21)$$

The subsequent quantum Hawking's evaporation can produced only via some collective excitations and formation, e.g., of a long chain of microscopic semi-closed worlds.

Classical analog of quantum black hole

Let us consider large ($m >> m_{pl}$) quantum black holes. The number of shells (both primary ones and created during collapse) is also very large, and one may hope to construct some classical continuous matter distribution that would mimic the properties of quantum black holes. First of all, we should translate the "no memory" state ($n = 0$ for all the shells) into "classical language". To do this let us rewrite the Eqn.(9) (energy constraint equation) for the shell, inside which there is some gravitating mass m_{in},

$$\sqrt{\dot{\rho}^2 + 1 - \frac{2Gm_{in}}{\rho}} - \sqrt{\dot{\rho}^2 + 1 - \frac{2Gm_{out}}{\rho}} = \frac{GM}{\rho} \tag{22}$$

and consider a turning point, ($\dot{\rho} = 0$, $\rho = \rho_0$):

$$\Delta m = m_{out} - m_{in} = M\sqrt{1 - \frac{2Gm_{in}}{\rho_0}} - \frac{GM^2}{2\rho_0}. \tag{23}$$

It is clear now that in order to make parameters of the shell (Δm and M) not depending on what is going on inside we have to put $m_{in} = a\rho_0$.

Our quantum black hole is in a stationary state. Therefore, a classical matter distribution should be static. We will consider a static perfect fluid with energy density ε and pressure p. A static spherically symmetric metric can be written as

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{24}$$

where ν and λ are functions of the radial coordinate r only. The relevant Einstein's equations are (prime denotes differentiation in r)

$$\begin{aligned} 8\pi G\varepsilon &= -e^{\lambda}(\frac{1}{r^2} - \frac{\lambda'}{r}) + \frac{1}{r^2}, \\ -8\pi Gp &= -e^{\lambda}(\frac{1}{r^2} - \frac{\nu'}{r}) + \frac{1}{r^2}, \\ -8\pi Gp &= -\frac{1}{2}e^{\lambda}(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu'\lambda'}{2}) \end{aligned} \tag{25}$$

The first of these equations can be integrated to yield

$$e^{-\lambda} = 1 - \frac{2Gm(r)}{r}, \tag{26}$$

where

$$m(r) = 4\pi \int_0^r \varepsilon r'^2 dr' \tag{27}$$

is the mass function, that must be identified with m_{in}. Thus, $m(r) = ar$, and

$$\varepsilon = \frac{a}{4\pi r^2}, \qquad e^{-\lambda} = 1 - 2Ga. \tag{28}$$

We can also introduce a bare mass function

$$M(r) = 4\pi \int_0^r \varepsilon e^{\frac{\lambda}{2}} r'^2 dr', \tag{29}$$

and from Eqn.(28) we get

$$M(r) = \frac{ar}{\sqrt{1-2Ga}} \tag{30}$$

The remaining two equations can now be solved for $p(r)$ and e^{ν}. The solution for $p(r)$ that has the correct nonrelativistic limit is

$$p(r) = \frac{b}{4\pi r^2}, \qquad b = \frac{1}{G}(1 - 3Ga - \sqrt{1-2Ga}\sqrt{1-4Ga}), \tag{31}$$

and for e^{ν} we have

$$e^{\nu} = Cr^{2G\frac{a+b}{1-2Ga}}. \tag{32}$$

The constant of integration C can be found from matching of the interior and exterior metrics at some boundary $r = r_0$. Let us suppose that $r > r_0$ the space-time is empty, so the interior should be matched to the Schwarzschild metric. Of course, to compensate the jump in pressure ($\Delta p = p(r_0) = p_0$) we must introduce some surface tension Σ. From matching conditions it follows that

$$\begin{aligned} C &= (1-2Ga) r_0^{-2G\frac{a+b}{1-2Ga}}, \\ e^{\nu} &= (1-2Ga)(\frac{r}{r_0})^{2G\frac{a+b}{1-2Ga}}, \\ \Delta p &= \frac{2\Sigma}{r_0} \end{aligned} \tag{33}$$

We would like to stress that the pressure p in our classical model is not real but only effective because it was introduce in order to mimic the quantum stationary states. We see, that the coefficient b in Eqn.(31) becomes a complex

number if $a > 1/4G$. Hence, we must require $a \leq 1/4G$, and in the limiting point we have the stiffest possible equation of state $\varepsilon = p$ It means also that hypothetical quantum collective excitations (phonons) would propagate with the speed of light and could be considered as massless quasi-particles. It is remarkable that in the limiting point we have $m(r) = M(r)/\sqrt{2}$ - the same relation as for the total and bare masses in the "no memory" state $n = 0$! The total mass $m_0 = m(r_0)$ and the radius r_0 in this case are related $m_0 = 4Gr_0$ - twice the horizon size.

Calculations of Riemann curvature tensor R^i_{klm} and Ricci tensor R_{ik} show that if $p < \varepsilon$ $(a \neq b)$ there is a real singularity at $r = 0$. But, surprisingly enough, both Riemann and Ricci tensors have finite limits at $r \to 0$, if $\varepsilon = p$ (a=b=1/4G). Therefore we are allowed to introduce the so-called topological temperature in the same way as for classical black holes. The recipe is the following. One should transform the space-time metric by the Wick rotation to the Euclidean form and smooth out the canonical singularity by the appropriate choice of the period for the imaginary time coordinate. The imaginary time coordinate is considered proportional to some angle coordinate. In our case the point $r = 0$ is already the coordinate singularity. The azimuthal angle ϕ has the period equal to π. Thus, all other angles should be periodical with the period π. The topological temperature is just the inverse of this period.

The easy exercise shows, that the temperature

$$T = \frac{1}{2\pi r_0} = \frac{1}{8\pi G m_0} = T_{BH} \tag{34}$$

exactly the same as the Hawking's temperature T_{BH} [5]! The very possibility of introducing a temperature provides us with the one-parameter family of models with universal distributions of energy density and pressure

$$\varepsilon = p = \frac{1}{16\pi G r^2}, \tag{35}$$

the parameter being the total mass m_0 or the size $r_0 = 4Gm_0$. It should be noted that the two-dimensional part of the metric obtained is nothing more but the Rindler's metric, and the null surface $r = 0$ serves as the event horizon.

We can now develop some thermodynamics for our model. First of all we should distinguish between global and local thermodynamic quantities. The

global quantities are those measured by a distant observer. He measures the total mass of the system m_0 and the black temperature $T_{BH} = T_\infty$ and does not know anything more. Let us assume that this observer is rather educated in order to recognize he is dealing with a black hole and to write the main thermodynamic relation

$$dm = TdS. \tag{36}$$

In this way he ascribes to a black hole some definite amount of entropy, namely, the Hawking-Bekenstein value

$$S = \frac{1}{4}\frac{(4\pi r_g)^2}{l_{pl}^2} = 4\pi G m_0^2 = 4\pi \left(\frac{m_0}{M_{pl}}\right)^2 \tag{37}$$

Moreover, if this observer is acquainted with, say, the book [32], he can learn from Chapter 3 that, using the Euclidean path integral technique, one can calculate a partition function for Schwarzschild black hole,

$$Z = \sum_n \exp\left(-\beta\varepsilon_n\right) = \exp\left(-\frac{\beta^2}{16\pi G}\right), \tag{38}$$

with β equal to the inverse of Hawking's temperature, $\beta = 1/T_H$, and derive the expectation value of the energy, in other words, the black hole mass,

$$m =< E >= -\frac{d}{d\beta}(\ln Z) = \frac{\beta}{8\pi G}. \tag{39}$$

Remembering then, that the free energy $F = -T\ln Z$ and $F =< E > -TS$, he can easily obtain for the entropy $S = \frac{1}{4}\frac{A_h}{l_{Pl}^2}$. Calculating the entropy in such a way, the observer builds some statistical background for the black hole thermodynamics. However, the obstacle in applying usual thermodynamical relations to essentially nonlocal (= global) objects, such as black holes, is that the corresponding extensive parameters, considered as thermodynamical potentials, are not the homogeneous first order functions of all the other extensive parameters. Indeed, the entropy S is a quadratic function of the mass (energy), and the free energy F is a function of the temperature alone (because there are no such extensive parameters like V (volume) and N ("particle" number) which would characterize a black hole thermo-equilibrium state).

The local observer who measures distributions of energy, pressure and local temperature is also rather educated and writes quite a different thermodynamic relation

$$\varepsilon(r) = T(r)s(r) - p(r) - \mu(r)n(r). \tag{40}$$

Here $\varepsilon(r)$ and $p(r)$ are energy density and pressure, $T(r)$ is the local temperature distribution, $s(r)$ is the entropy density, $\mu(r)$ is the chemical potential, and $n(r)$ is the number density of some (quasi)"particles". For the energy density and pressure the local observer gets, of course, the relation (35), and for the temperature - the following distribution

$$T(r) = \frac{1}{\sqrt{2}\pi r}, \tag{41}$$

which is compatible with the law $T(r)e^{\frac{\nu}{2}} = const$ and the boundary condition $T_\infty = T_{BH}$. Such a distribution is remarkable in that if some outer layer of our perfect fluid would be removed, the inner layers would remain in thermodynamic equilibrium. And what about the entropy density? Surely, the local observer is unable to measure it directly or calculate without knowing the microscopic structure of the system, but he can receive some information concerning the total entropy from the distant observer. This information and the measured temperature distribution (41) allows him to deduce that

$$s(r) = \frac{1}{8\sqrt{2}Gr} \tag{42}$$

and

$$s(r)T(r) = \frac{1}{16\pi G r^2} \tag{43}$$

It is interesting to note that in the main thermodynamic equation the contribution from the pressure is compensated exactly by the contribution from the temperature and entropy. It is noteworthy to remind that the pressure in our classical analog model is of quantum mechanical origin as well as the black hole temperature. And what is left actually is the dust matter we started from in our quantum model, namely,

$$\varepsilon = \mu n = \frac{1}{16\pi G r^2} \tag{44}$$

We may suggest now that the quantum black hole is the ensemble of some collective excitations, the black hole phonons, and $n(r)$ is just the number density of such phonons.

Knowing equation of state, $\varepsilon = p$, we are able to construct all the thermodynamical potentials for our system. As an example we show here how to calculate the energy as a function of the entropy S, and the number particles N (the extensive thermodynamical variables E, S, V,N are denoted by capital letters and assumed to have macroscopic but small enough values). By the first law of thermodynamics

$$dE = TdS - pdV + \mu dN \tag{45}$$

where $T = \left.\frac{\partial E}{\partial S}\right|_{V,N}$ is a temperature $p = -\left.\frac{\partial E}{\partial V}\right|_{S,N}$ is a pressure, and $\mu = \left.\frac{\partial E}{\partial N}\right|_{S,V}$ is a chemical potential. The energy is additive with respect to the particle number N, hence, $E = Nf(x,y)$ where $x = \frac{S}{N}$ and $y = \frac{N}{V}$. Since $\varepsilon = \frac{E}{V} = yf(x,y)$ and $p = y^2\frac{\partial f}{\partial y}$ from the equation of state we obtain

$$\begin{aligned} f = \alpha(x) = n\alpha(x) \\ \varepsilon = p = n^2\alpha(x) \end{aligned}$$

Further,

$$\begin{aligned} T = n\alpha'(x) \\ \mu = n(2\alpha - x\alpha') \end{aligned}$$

But, in any static gravitational field $T = T_0/\sqrt{g_{00}}$ and $\mu = \mu_0/\sqrt{g_{00}}$, so $\mu = \gamma_0 T$, where γ_0-some numerical factor. Thus,

$$\begin{aligned} 2\alpha - x\alpha' = \gamma_0\alpha', \\ \alpha(x) = C_0(\gamma_0 + x)^2 \end{aligned}$$

where C_0 is a constant of integration. It is easy to see that $p/T^2 = 1/4C_0$. In our specific model $p/T^2 = \pi/8G$, so $C_0 = 2G/\pi$. Moreover, because of the relation $\varepsilon = p = Ts = \mu n$ we know that the free energy $F = E - TS$ is numerically zero. From this we have for the entropy

$$S = \gamma_0 N \tag{46}$$

The black hole entropy equals one fourth of the dimensionless horizon area, and from this we recover the famous Bekenstein-Mukhanov mass spectrum

$$m = \sqrt{\frac{\gamma_0}{4\pi}}\sqrt{N} m_{pl} \tag{47}$$

Note, that our model gives for the free energy an expression quite different from that obtained by the use of global thermodynamics. In the latter $F = \frac{1}{16\pi G T}$ which is numerically equal to $\frac{m}{2}$. In our case

$$F = F(T, V, N) = \gamma_0 N T - \frac{V T^2}{4 C_0}, \tag{48}$$

but the relation (39) is nevertheless fulfilled.

In principle, we can even calculate the remaining unknown coefficient γ_0 using the phonon model. Indeed, since $F = 0$, the partition function

$$Z = \sum_n e^{-\frac{\varepsilon_n}{T}} = 1 \tag{49}$$

Let us assume that our gravitational phonons have the equidistant energy spectrum $\varepsilon_n = \omega n$. Then, $\omega_n - \omega_{n-1} = dE = \omega dN$, $(dN = 1)$. Note, that on the static gravitational field the ratio $\frac{\omega}{T}$ is an invariant. Therefore, we can use dm and T_{BH} (e.i., the increase in the total mass m and the Hawking's temperature) instead of local quantities dE and T. Then,

$$\frac{dm}{T_{BH}} = 8\pi G m dm = dS = \gamma_0 dN = \gamma_0, \tag{50}$$

$$\tag{51}$$

$$Z = \sum_n e^{-\frac{\omega n}{T}} = \frac{e^{-\gamma_0}}{1 - e^{-\omega_0}} = 1, \tag{52}$$

$$\tag{53}$$

$$\gamma_0 = \ln 2 \tag{54}$$

This just the value advocated by J.Bekenstein and V.Mukhanov in the spirit of information theory. If we accept the harmonic oscillator spectrum $\varepsilon_n = \omega(n + \frac{1}{2})$ we would obtain $\gamma_0 = 2\ln\frac{\sqrt{5}+1}{2} \approx 1$.

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